A MATHS LESSON

Equivalent Fractions

Different names for the same amount

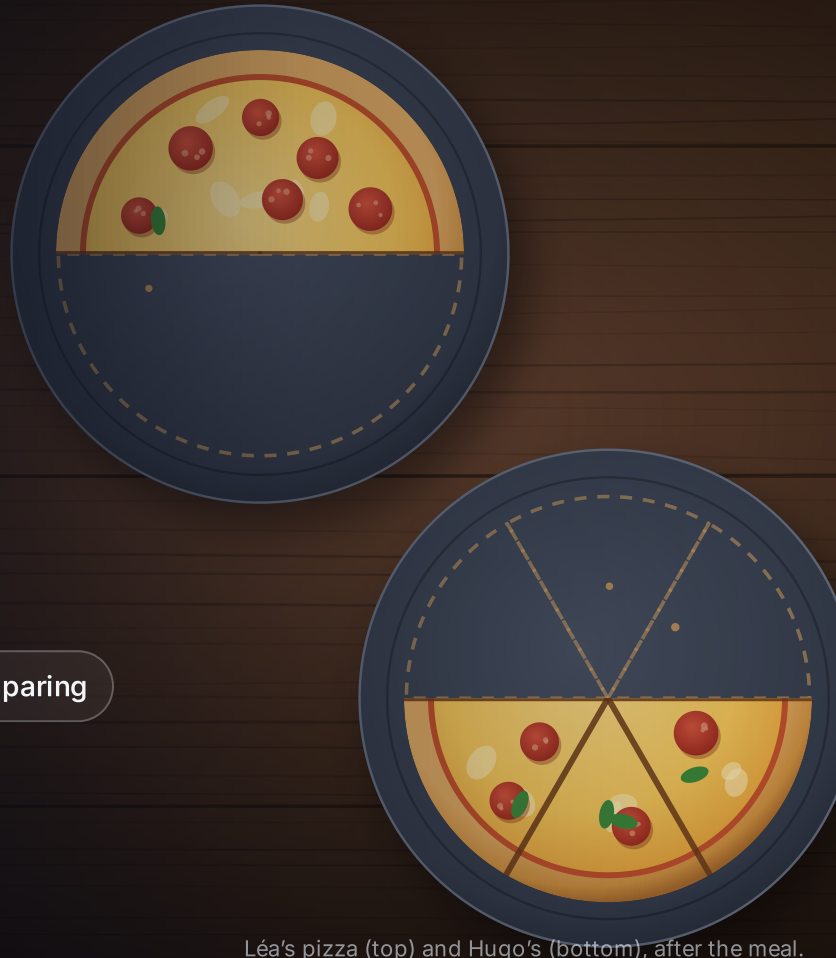
Léa eats 1 slice of a pizza cut into 2. Hugo eats 3 slices of a pizza just like it, cut into 6. Who ate more?

1 One amount, many names

2 Making them

3 Spotting and comparing

4 What they are for



Léa's pizza (top) and Hugo's (bottom), after the meal.

1 · ONE AMOUNT, MANY NAMES

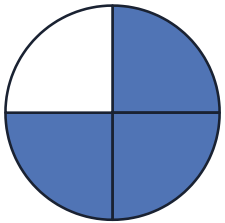
What a fraction says

A fraction names a part of a whole cut into **equal parts**. In $\frac{3}{4}$, the whole is cut into 4 equal parts, and 3 of them are taken: three quarters.

$$\frac{3}{4}$$

3, the numerator
how many parts are taken

4, the denominator
how many equal parts the whole is cut into





REMEMBER

The parts must be equal. A pizza cut into 4 slices of different sizes is not cut into quarters.

? **In $\frac{5}{8}$, how many parts? How many taken?**

The whole is cut into 8 equal parts, eighths, and 5 of them are taken.

? **One egg out of a dozen?**

$\frac{1}{12}$: a dozen is 12 eggs.

? **And $\frac{4}{4}$?**

All 4 quarters: the whole. $\frac{4}{4} = 1$, like $\frac{2}{2}$, $\frac{3}{3}$ or $\frac{10}{10}$.

1 · ONE AMOUNT, MANY NAMES

The same amount, other names

Léa and Hugo ate the same amount of pizza: 1 slice out of 2 is half the pizza, and so are 3 slices out of 6. The fractions $\frac{1}{2}$ and $\frac{3}{6}$ are **equivalent**: they name the same amount.

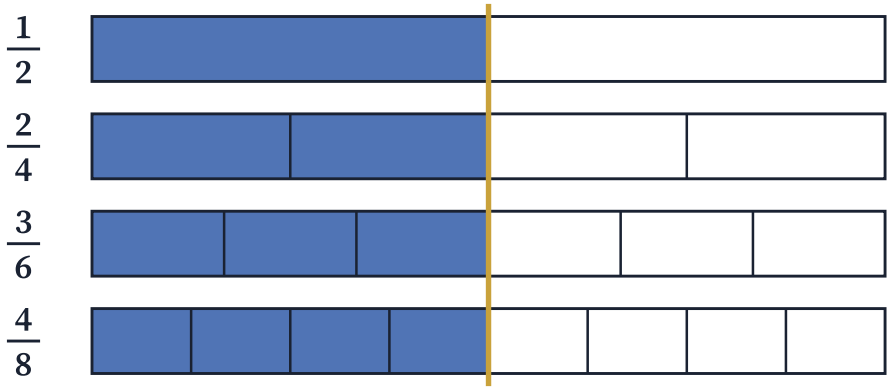
$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

? **Half, with 10 parts?**

$\frac{5}{10}$: 5 parts out of 10.

? **Half, with 100 parts?**

$\frac{50}{100}$.



On each bar, the coloured part stops at the same place, in the middle. The parts are smaller, but there are more of them.

REMEMBER

Equivalent fractions are equal: they are the same number, written differently, and we write $\frac{1}{2} = \frac{3}{6}$. A fraction has infinitely many: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \ldots = \frac{50}{100} = \ldots$

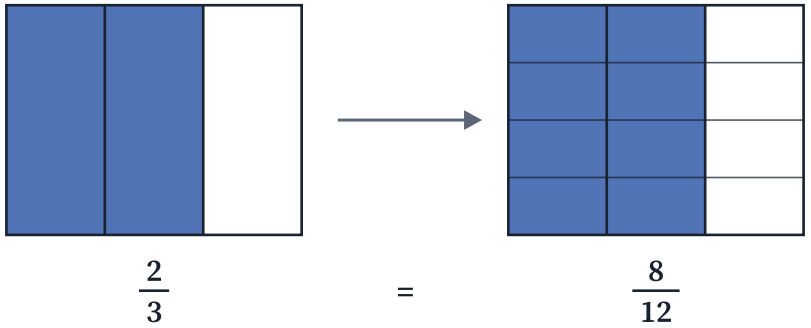
Multiply the top and the bottom

Cut each third of $\frac{2}{3}$ into 4: there are 4 times as many parts in all ($3 \times 4 = 12$), and 4 times as many coloured parts ($2 \times 4 = 8$).

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Multiply the numerator and the denominator by the same number: the fraction keeps its value.

Multiplying the top and the bottom by 4 is multiplying the fraction by $\frac{4}{4}$, that is, by 1: its value does not change.



? $\frac{3}{5} = \frac{?}{10}$

10 = 5 × 2, so 3 × 2 = **6**.

? $\frac{1}{4} = \frac{5}{?}$

5 = 1 × 5, so 4 × 5 = **20**.

? $\frac{2}{7} = \frac{?}{21}$

21 = 7 × 3, so 2 × 3 = **6**.

? $\frac{5}{6} = \frac{20}{?}$

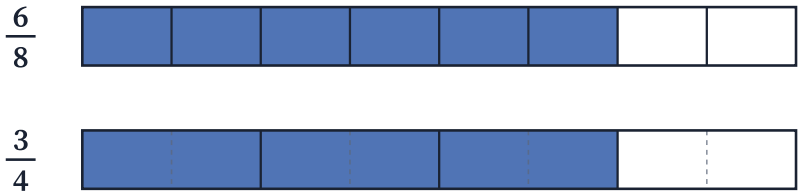
20 = 5 × 4, so 6 × 4 = **24**.

Simplifying

The other way round, put the parts back together. In $\frac{6}{8}$, glue the eighths two by two: they become quarters, and 6 coloured eighths become 3 coloured quarters.

$$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

Divide the numerator and the denominator by the same number, one that divides both.



The divisibility rules help. Both even? Divide by 2. Both end in 0 or 5? By 5. Both digit sums multiples of 3? By 3. Step by step: $\frac{18}{24}$ = $\frac{9}{12}$ (÷ 2) = $\frac{3}{4}$ (÷ 3).

? Simplify $\frac{10}{15}$

Both end in 0 or 5: ÷ 5 gives $\frac{2}{3}$.

? Simplify $\frac{12}{16}$

÷ 2: $\frac{6}{8}$, ÷ 2 again: $\frac{3}{4}$. Or ÷ 4 straight away.

? Simplify $\frac{9}{27}$

÷ 9: $\frac{1}{3}$, since 27 = 3 × 9.

? Simplify $\frac{14}{21}$

Both are in the 7 times table: ÷ 7 gives $\frac{2}{3}$.

The simplest form

When only 1 divides both the numerator and the denominator, the fraction cannot be simplified any more: it is in its **simplest form**. $\frac{3}{4}$ is in simplest form; $\frac{6}{8}$ is not.

In one go, with the GCD

Divide both by their greatest common divisor.

For $\frac{24}{36}$: GCD(24, 36) = 12, and 24 ÷ 12 = 2, 36 ÷ 12 = 3. So $\frac{24}{36} = \frac{2}{3}$, in a single step.

▶ **Euclid's algorithm, for big numbers**

Little by little

Divide by small numbers, as long as you can.

$\frac{24}{36} = \frac{12}{18}$ (÷ 2) $= \frac{6}{9}$ (÷ 2) $= \frac{2}{3}$ (÷ 3). More steps, the same result.

? $\frac{45}{60}$ **in simplest form**

GCD(45, 60) = 15: $\frac{3}{4}$.

? $\frac{32}{48}$ **in simplest form**

GCD(32, 48) = 16: $\frac{2}{3}$.

? $\frac{35}{49}$ **in simplest form**

GCD(35, 49) = 7: $\frac{5}{7}$.

? $\frac{84}{120}$ **in simplest form**

GCD(84, 120) = 12: $\frac{7}{10}$.

Watch out for traps

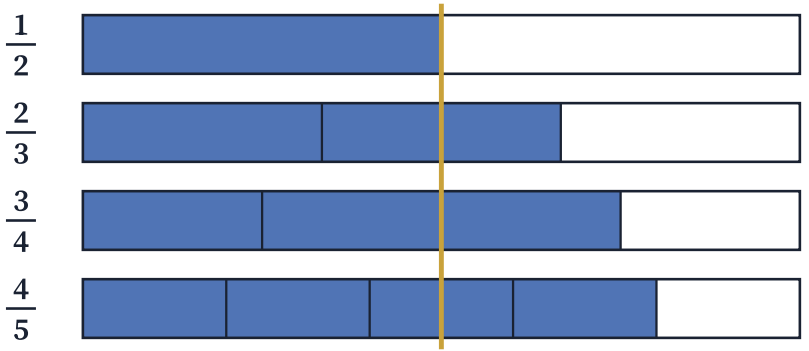
Only multiplying or dividing the top and the bottom by the same number gives an equivalent fraction. Two common mistakes:

ADDING THE SAME NUMBER

Add 1 above and below $\frac{1}{2}$, and you get $\frac{2}{3}$, which is bigger. Do it again: $\frac{3}{4}$, then $\frac{4}{5}$… the fraction grows every time.

CHANGING ONLY ONE NUMBER

$\frac{2}{3} \rightarrow \frac{4}{3}$: only the top was multiplied by 2, and the fraction doubled. Both numbers, always!



? Is $\frac{3}{5}$ equal to $\frac{4}{6}$?

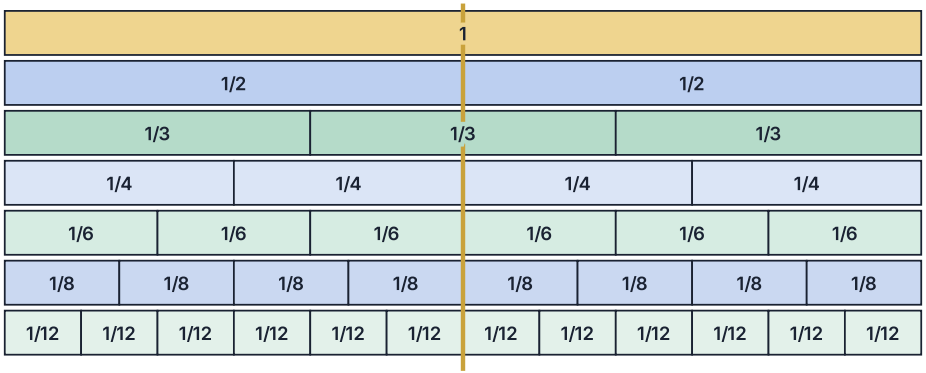
No: 1 was added above and below. In thirtieths, $\frac{3}{5} = \frac{18}{30}$ but $\frac{4}{6} = \frac{20}{30}$.

? $\frac{16}{64}$: cross out the two 6s, and you get $\frac{1}{4}$. Is that right?

$\frac{16}{64} = \frac{1}{4}$ indeed (÷ 16), but by luck. Try it on $\frac{13}{39}$: crossing out the 3s gives $\frac{1}{9}$, yet $\frac{13}{39} = \frac{1}{3}$ (÷ 13).

The fraction wall

Each row of the wall is a whole of the same length, cut into equal bricks. Bricks that end at the same place show equivalent fractions: the gold line marks $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12}$.



? **Equal to $\frac{1}{3}$?**

$\frac{2}{6}$ and $\frac{4}{12}$: two sixths and four twelfths end where the first third ends.

? **Equal to $\frac{2}{3}$?**

$\frac{4}{6}$ and $\frac{8}{12}$.

? **Equal to $\frac{3}{4}$?**

$\frac{6}{8}$ and $\frac{9}{12}$.

Equivalent or not?

Three ways to know whether two fractions are equivalent without drawing anything, here with $\frac{6}{9}$ and $\frac{10}{15}$:

Simplify both

If they come to the same simplest form, they are equivalent.

$\frac{6}{9} = \frac{2}{3}$ (÷ 3) and $\frac{10}{15} = \frac{2}{3}$ (÷ 5).

Cross products

$\frac{a}{b}$ and $\frac{c}{d}$ are equivalent when $a \times d = b \times c$.

$6 \times 15 = 90$ and $9 \times 10 = 90$: equal.

The same denominator

Write both over a common denominator: then the numerators must be equal.

$\frac{6}{9} = \frac{30}{45}$ and $\frac{10}{15} = \frac{30}{45}$.

? $\frac{4}{6}$ **and** $\frac{6}{9}$**?**

Yes: $4 \times 9 = 36 = 6 \times 6$. Both are $\frac{2}{3}$.

? $\frac{3}{4}$ **and** $\frac{9}{16}$**?**

No: $3 \times 16 = 48$, but $4 \times 9 = 36$.

? $\frac{15}{25}$ **and** $\frac{9}{15}$**?**

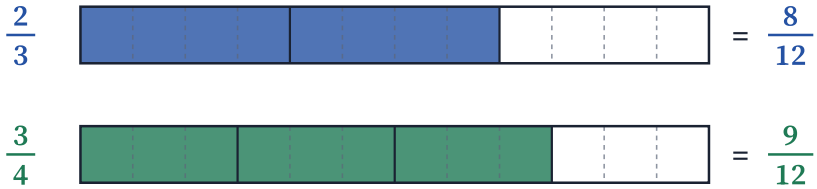
Yes: both simplify to $\frac{3}{5}$. Check: $15 \times 15 = 225 = 25 \times 9$.

Comparing fractions

Which is bigger, $\frac{2}{3}$ or $\frac{3}{4}$? Their parts are not the same size: write both fractions over a common denominator, the LCM of the denominators, then compare the numerators.

LCM(3, 4) = 12: $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$.

$\frac{8}{12} < \frac{9}{12}$, so $\frac{2}{3} < \frac{3}{4}$



TWO SHORTCUTS

Same denominator: the bigger numerator wins, $\frac{5}{7} > \frac{3}{7}$.

Same numerator: the smaller denominator wins, as its parts are bigger, $\frac{3}{5} > \frac{3}{8}$.

? $\frac{3}{5}$ or $\frac{2}{3}$?

LCM(5, 3) = 15: $\frac{9}{15} < \frac{10}{15}$. $\frac{2}{3}$ is bigger.

? $\frac{5}{6}$ or $\frac{7}{9}$?

LCM(6, 9) = 18: $\frac{15}{18} > \frac{14}{18}$. $\frac{5}{6}$ is bigger.

? $\frac{4}{7}$ or $\frac{4}{9}$?

The same numerator: sevenths are bigger than ninths, so $\frac{4}{7}$.

4 · WHAT THEY ARE FOR

Adding and subtracting

Only parts of the same size can be added. To add $\frac{2}{3}$ and $\frac{1}{4}$, first write them over a common denominator: as equivalent fractions.

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$$

$$\frac{5}{6} - \frac{1}{2} = \frac{5}{6} - \frac{3}{6} = \frac{2}{6} = \frac{1}{3}$$

Equivalent fractions are used twice: to get the same denominator, then to simplify the answer.

▶ Adding Fractions

The LCM lesson

? $\frac{1}{2} + \frac{1}{3}$?

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

? $\frac{3}{4} - \frac{1}{8}$?

$\frac{6}{8} - \frac{1}{8} = \frac{5}{8}$.

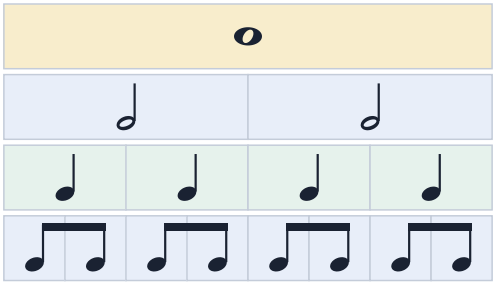
? $\frac{1}{6} + \frac{1}{3}$?

$\frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$.

In everyday life

Music

A whole note lasts as long as 2 half notes, 4 quarter notes or 8 eighth notes.



A half note is $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$ of a whole note, and two eighth notes take the place of one quarter note: $\frac{2}{8} = \frac{1}{4}$.

The clock

An hour has 60 minutes. A quarter of an hour is 15 minutes, since $\frac{1}{4} = \frac{15}{60}$; three quarters of an hour, 45 minutes; and 20 minutes is a third of an hour: $\frac{20}{60} = \frac{1}{3}$.

Cents and percentages

The 25-cent coin is called a quarter: $\frac{1}{4}$ of a dollar is $\frac{25}{100}$. A percentage is a fraction out of 100: $\frac{3}{4} = \frac{75}{100} = 75\%$.

REVIEW

Test yourself

1 **Complete: $\frac{3}{8} = \frac{?}{24}$**

24 = 8 × 3, so 3 × 3 = **9**: $\frac{3}{8} = \frac{9}{24}$.

2 **The simplest form of $\frac{36}{48}$?**

GCD(36, 48) = 12: $\frac{36}{48} = \frac{\mathbf{3}}{\mathbf{4}}$.

3 **Are $\frac{6}{10}$ and $\frac{9}{15}$ equivalent?**

Yes: both are $\frac{3}{5}$. Check: 6 × 15 = 90 = 10 × 9.

4 **Which is bigger, $\frac{5}{8}$ or $\frac{2}{3}$?**

LCM(8, 3) = 24: $\frac{5}{8} = \frac{15}{24}$ and $\frac{2}{3} = \frac{16}{24}$. So $\frac{\mathbf{2}}{\mathbf{3}}$, only just.

5 **How much is $\frac{1}{3} + \frac{1}{4}$?**

$\frac{4}{12} + \frac{3}{12} = \frac{\mathbf{7}}{\mathbf{12}}$.

6 **40 minutes: what fraction of an hour, in simplest form?**

$\frac{40}{60} = \frac{\mathbf{2}}{\mathbf{3}}$ of an hour (÷ 20).

TO SUM UP

The big ideas

1. **Equivalent fractions** name the same amount, like $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$: they are equal.

2. Multiply or divide the numerator and the denominator by the **same number**; never add. Dividing by the GCD gives the **simplest form**.

3. Over a **common denominator**, the LCM of the denominators, fractions can be compared, added and subtracted.

Practise — endless questions, with an explanation after each mistake, in the Practice Room:

▶ Equivalent fractions | Simplifying a fraction

Go further

Adding Fractions | Fraction Express | The LCM lesson | Euclid's Algorithm